Uncertainty versus Rigidities - What Is the Main Driver of Deviations from Full Information Rational Expectations?∗

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Abstract

In a full information rational expectations world, agents immediately respond to shocks. In reality however, there are often large delays after shocks. There is an extensive literature on the information flow that attempts to explain this. Some of the literature explains the delay with uncertainty about the future and some explain it with information rigidity. This paper estimates the extent of both types of delays at the aggregate and individual level with a novel approach. The results show that while the delay is substantial, it is largely driven by uncertainty.

JEL: C53, D84, E17, E31, E37

Keywords: Information rigidity, Bloomberg Survey, rational expectations, SPF, noisy information, sticky information, Calvo updating, state dependent updating, uncertainty

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1 Introduction

This paper estimates the deviations from full information rational expectations and disentangles the role of uncertainty about the future from information rigidity in the US economy. Novel approaches are utilized to estimate information rigidity and the overall delay.

Deviations from full information rational expectations can cause delayed responses to policy shocks. An example is the US experience in the late 70s, when inflation expectations based on the forecasts in the Survey of Professional Forecasts (SPF) only fell below 5% year over year more than two years after Volcker started to dramatically increase the interest rate to signal the tough stance on inflation.\footnote{While forecasts might not be equivalent to expectations if the loss functions are not symmetric as shown in Messina et al. (2015) for the Federal Reserve, this does not appear to be a large issue in a broader set of forecasters as shown in Bürgi (2017).}

Because this delay can be substantial, it is important for policy makers to know what causes the delay, how long it is and to what extent the delay can be influenced. This paper mainly addresses the first two questions by estimating the contribution to the delay stemming from information rigidity and uncertainty using a novel approach.\footnote{Information rigidity in this paper follows the structure of models presented in Rotemberg (1982), Calvo (1983), Mankiw and Reis (2002), Woodford (2002) or Sims (2003) and uncertainty models presented in Bloom (2009), Lahiri and Sheng (2010), Jurado et al. (2015), Baker et al. (2016) or Ozturk and Sheng (2017).}

This paper shows how several competing models of information rigidity can be combined into one, allowing the estimation of the joint rigidity effect. It is shown that uncertainty is needed for previous approaches to correctly estimate the contribution of information rigidity to the deviations from full information rational expectations at the aggregate level (e.g. Coibion and Gorodnichenko (2015), Dovern et al. (2015) or Loungani et al. (2013)).

The new estimates show that the contribution of information rigidity to the
overall delay is sizable at the short horizons while the contribution of uncertainty is large at all horizons. Together, these estimates suggest that uncertainty is the main driver of the deviations from full information rational expectations and particularly so at long horizons. In contrast to much of the previous literature, this paper estimates the deviations at both the aggregate and individual level. The individual level estimates for the overall delay are very close to the aggregate level ones. However, information rigidity appears to be less pronounced and contribute less to the overall delay at the individual level relative to the aggregate level. This provides further insights into the information formation process.

In addition to the before mentioned literature on estimating information rigidity, this paper is related to the micro level estimates (e.g. Dräger and Lamla (2012), Andrade and Bihan (2013) and Sheng and Wallen (2014)) and the nominal rigidity literature both at the aggregate and individual level (e.g. Khan and Zhu (2006), Döpke et al. (2008), Klenow and Malin (2010) or Nakamura and Steinsson (2013)).

The remainder of this paper is structured as follows: The next section will explain how uncertainty affects the optimal prediction of agents. Subsequently, a general model of information rigidity is introduced and combined with the uncertainty model. It will be shown that this overall model encompasses a combination of individual rigidity models. It is then detailed how to estimate both rigidity and the overall deviation from full information rational expectations. The subsequent section will detail the data used for the analysis, followed by the section on the empirical analysis, where it will be shown that there are extensive deviations from full information rational expectations and that the main driver of these deviations is uncertainty. The last section concludes.
2 Optimal prediction under uncertainty

Under full information rational expectations, agents predict the underlying variable without error. That is,

\[ F_{jt,t-i} = A_t \forall i, t, j \] (1)

If agents do not have full information but there is no information rigidity, every prediction they make will have some error attached to them. The expected error of the predictions made is uncertainty as for example defined in Lahiri and Sheng (2010).

A convenient way to model this is to assume that periodically, agents receive new information about the future. This new information will not be perfectly accurate and have some error \( \nu_{jt-i} \) with variance \( \sigma^2_{\nu_{jt-i}} \) attached to it. This error can also differ across agents:

\[ x_{jt-i} = A_t + \nu_{jt-i} \] (2)

Assuming further that the new information agents receive is independent of the information they already have from the past, agents will optimally weight the old and new information using Bates and Granger (1969) optimal weights. This setup is very similar to a noisy information setup in Lucas (1972), Finn E. Kydland (1982), Woodford (2002), and Sims (2003) and thus the Kalman filter.\(^3\) However, in contrast to the implementation in Coibion and Gorodnichenko (2015), it is assumed that agents can observe the current state of the underlying variable and the only uncertainty they face is about the future. The agents then receive noisy signals about the future state such that there is no information rigidity in this model.\(^4\)

Without any information rigidity but with uncertainty, agents will optimally weight their old prediction and the new information to obtain the best possible

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\(^3\) A Kalman filter implementation is detailed in Appendix B

\(^4\) An alternative way to motivate this model would be that agents learn about the future state as described for example in Foster and Frierman (1990)
new prediction. That is
\[ F_{jt,t-i} = \frac{\sigma^2_{\varepsilon t-1}}{\sigma^2_{\varepsilon t-1} + \sigma^2_{\nu t-i}} x_{jt-i} + \frac{\sigma^2_{\nu t-i}}{\sigma^2_{\varepsilon t-1} + \sigma^2_{\nu t-i}} F_{jt,t-i-1}, \]  
(3)
where \( \sigma^2_{\varepsilon t-1} \) is the variance of the error of the previous prediction \( A_t - F_{jt,t-i-1} \).

This is also the underlying assumption of the Nordhaus (1987) test which checks, whether agents put this optimal weight on past and new information (see Appendix A).\(^5\) It is easy to see that uncertainty causes the auto-correlation because \( \sigma^2_{\nu t-i} \rightarrow 0 \) as the signal noise and thus uncertainty decreases. In turn as uncertainty increases, the auto-correlation converges to one.

Equation 3 cannot be directly estimated, as all variables are correlated with \( A_t \) and the error of the new information is unobserved. Given the optimal weights on old and new information, running a regression with the prediction on the left hand side and the most recent forecast revision on the right hand side should not lead to any significant coefficient. Indeed, assuming \( \lambda \) is the weight put on old information, the coefficient would become
\[ \hat{\beta} = \frac{-\sigma^2_{\varepsilon t} + \frac{\lambda}{1-\lambda} \sigma^2_{\nu t-i} \sigma^2_{\varepsilon t-1}}{\sigma^2_{\varepsilon t} + \sigma^2_{\nu t-i} \sigma^2_{\varepsilon t-1}} = 0, \]
(4)
because \( \frac{\lambda}{1-\lambda} = \frac{\sigma^2_{\nu t-i}}{\sigma^2_{\varepsilon t-1}} \) from equation 3.

There is also a very closely related approach to test whether agents put the optimal weight on old and new information as detailed in Dovern et al. (2015). That is, a regression is run with the most recent revision on the left and a previous revision on the right. While their approach has the advantage that the

\(^5\)Another commonly used approach in the literature to describe uncertainty is to say that the prediction error is the sum of future shocks plus some idiosyncratic error (e.g. Lahiri and Sheng (2010)). This approach is not taken here, because it would cause the covariance between two successive prediction errors to explode as the horizon increases instead of converging to the variance of the underlying variable. Even more concerning, regressing two prediction errors with different horizons on each other requires the slope coefficient to be between zero and one, no matter, which of the two errors is on the right hand side and which one on the left.
actual observation is not necessary for their estimation, the resulting coefficient and assumptions are the same.  

3 Information rigidity

There are several standard models of information rigidity: sticky information in Mankiw and Reis (2002), Calvo (1983) updating or state dependent updating as in Rotemberg (1982). All models assume that without the rigidity, the optimal prediction of the value of the underlying variable $A$ in period $t$, predicted by agent $j$ in period $t-i$ is

$$F_{jt,t-i} = A_t + \nu_{t-i}, \quad (5)$$

where $\nu_{t-i}$ is the rational expectations error, which is the same across agents. With the rigidity in place, either agents do not always update to this optimal value due to a fixed cost to updating (e.g. the sticky information model or a Calvo updating model) or they do not update all the way due to a cost proportional to the size of the update (e.g. the state dependent model with menu cost). Both these types of costs cause agents new prediction to put some weight on the old prediction. This takes the form

$$F_{jt,t-i} = (1 - \lambda)(A_t + \nu_{t-i}) + \lambda F_{jt,t-i-1}, \quad (6)$$

where $\lambda$ is the weight put on the old prediction. To obtain the equation for the aggregate level, it is necessary to average individual predictions, which removes the index $j$.

The expression can be rearranged to a Nordhaus (1987) test, where the prediction error of the most recent prediction is equal to the most recent revision times the ratios of the weight on the old prediction and the weight on the optimal

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6While a bit more detail is provided in Appendix A, the exact expression is omitted here, as it can easily be calculated based on the derivations in the appendix.
current prediction:

\[ A_t - F_{t,t-i} = \frac{\lambda}{1-\lambda} (F_{t,t-i} - F_{t,t-i-1}) - \nu_{t-i}; \quad i = 0, 1, ..., 3. \] (7)

For the sticky information and Calvo model, there is an issue with estimating equation 7 at the individual level. In those models, information rigidity is caused by agents not updating predictions every period and the \( \lambda \)s are probabilities. This causes the estimate at the individual level to be biased downward without any instrument. Equation 7 can thus only lead to an unbiased estimate at the aggregate level for the sticky information and Calvo model.

### 3.1 Which rigidity model is the correct one?

Since there are small differences between the models at least at the individual level, it is important to determine, which model is the correct one. This section will show that all models discussed have some merit to them. At the same time, some basic assumptions of all models can easily be rejected. Fixing the flaws of the models will make the models more similar to each other and a combination of the models becomes likely. To estimate the models correctly once uncertainty is introduced, it is important to show the implications of a combined model on the overall rigidity estimation.

Under a strict sticky information or Calvo updating model, all agents update to the same new prediction. This assumption is trivially violated in the data. However, allowing agents to updates their predictions multiple times in between observing them could mitigate this. Every prediction would have different vintages. As different vintages will have different shocks, spreading updates this way causes every agent to have a different prediction, omitting this issue.

Alternatively, agents could behave strategically in the sticky information model (similar to the one discussed in Ehrbeck and Waldmann (1996)). All agents that gain new information have the same new information set. This implies that the only difference between agents that have a more accurate predic-
tion of the underlying variable relative to agents with a less accurate prediction is how often they obtain new information. This together with the difficulty in determining better and worse forecasters due to their high correlation as discussed in Birgi and Sinclair (2017) or Davies and Lahiri (1995) could lead to agents updating their predictions every period. This can be achieved by adding an iid error to every prediction, which in turn causes agents that update their forecasts to have different predictions. Note that this second fix only works in the sticky information model and not the Calvo updating model.

State dependent updating assumes that, all agents always make the same new prediction, which is again trivially violated. This can be mitigated by assuming agents do not have the same update cost. This way, all agents will have different predictions.

The more problematic parts of the models are mitigated, the more alike the models become. For example, with the additional assumptions added to the standard model, almost all agents update every period and have different predictions. Also, every model has its merit as there is at least anecdotal evidence regarding causes for the forms of rigidity described. Together, this makes a combination of models more likely than any individual standard model. Under the combination assumption for two models, the common equation 6 across models is replaced by the equation

\[ F_{jt,t-i} = (1 - \lambda)(1 - \rho)(A_t + \nu_{jt-i}) + (\lambda + \rho(1 - \lambda))F_{jt,t-i}. \]  

which provides only the coefficient \((\lambda + \rho(1 - \lambda))\) instead of \(\lambda\) and \(\rho\) separately. This combination also works with more than two models and will allow to estimate the overall rigidity of a combined model. Indeed, it even addresses models not discussed explicitly above, provided they cause a positive weight on past predictions. For example rounding could be a friction that causes the same pattern but was not addressed above. If the implied revision is too small, the rounded revision might be an unchanged prediction.
Given this is a combination of models, the assumptions required for an unbiased estimate are the same as in the previous section. In addition, a combination cannot be estimated at the individual level because of the before mentioned issue affecting the Calvo and sticky information models.

3.2 Combining uncertainty and rigidity

The uncertainty and rigidity models described so far need to be combined to obtain the overall deviation from full information rational expectations. In the general rigidity model above, the optimal prediction without rigidity is then replaced by the prediction based on equation 3. The reduced form

\[ F_{jt, t-i} = (1 - \lambda)(A_t + \nu_{jt-i}) + \lambda F_{jt, t-i-1}, \]

where \( A_t + \nu_{jt-i} \) is the rational expectation prediction is replaced by the optimally weighted prediction under uncertainty, which is

\[ F_{jt, t-i} = (1 - \lambda) \left[ \frac{\sigma^2_{\epsilon_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}} (A_t + \nu_{jt-i}) + \frac{\sigma^2_{\nu_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}} F_{jt, t-i-1} \right] + \lambda F_{jt, t-i-1} \]

\[ (10) \]

\[ F_{jt, t-i} = (1 - \lambda) \frac{\sigma^2_{\epsilon_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}} (A_t + \nu_{jt-i}) + \frac{\sigma^2_{\nu_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}} + \lambda \frac{\sigma^2_{\epsilon_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}} F_{jt, t-i-1} \]

\[ (11) \]

The weight of the past prediction in the new one is thus composed of the rigidity term \( \frac{\lambda \sigma^2_{\epsilon_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\epsilon_{t-1}}} \) and the uncertainty term \( \frac{\sigma^2_{\nu_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}} \). This also implies that just estimating \( \lambda \) will not provide the contribution of the rigidity to the overall deviation from full information rational expectations. This can be rearranged to have the Nordhaus type form

\[ A_t - F_{jt, t-i} = \hat{\beta}(F_{jt, t-i} - F_{jt, t-i-1}) - \nu_{t-i}; \quad i = 0, 1, ..., 3, \]

where

\[ \hat{\beta} = \frac{\sigma^2_{\nu_{t-1}} + \lambda \sigma^2_{\epsilon_{t-1}} - (1 - \lambda) \sigma^2_{\nu_{t-1}}}{(1 - \lambda)(\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}})} = \frac{\lambda}{1 - \lambda^2} \]

\[ (13) \]
For a more detailed derivation, see Appendix A. This is the same setup as used in Coibion and Gorodnichenko (2015) and Andrade and Bihan (2013). This implies that if the information rigidity measured by $\lambda$ is 0, the coefficient should take a value of 0 as well. Any positive coefficient indicates that there is rigidity. To obtain the exact amount of rigidity, a simple transformation is necessary.\(^7\)

### 3.3 Estimating uncertainty and rigidity

So far, the paper showed that the commonly used estimation of information rigidity does not estimate the contribution of information rigidity to the overall deviation from full information rational expectations. This section shows how the overall deviation from full information rational expectations can be estimated as well as the contribution of uncertainty and information rigidity, provided agents optimally weight old and new information.

Equation 11 showed the relationship between current and past predictions. This equation cannot be directly estimated, because the noise of the new information cannot be observed. However, it is possible to transform the equation to properly estimate the weight on the old prediction and thus old information.

If equation 11 is subtracted from the actual value, the equation

$$
\varepsilon_{jt-i} = \gamma \varepsilon_{jt-i-1} - (1 - \gamma) \nu_{jt-i}; \quad i = 0, 1, ..., 3
$$

\hspace{1cm} (14)

is obtained, where $\varepsilon_{jt-i}$ is the prediction error of the prediction made in period $t-i$ and $\gamma$ the coefficient of interest. This equation can be estimated without bias as long as $\nu_{t-i}$ is uncorrelated with $\varepsilon_{jt-i-1}$. This holds by assumption, since it was assumed that the noise of the new information was iid and thus uncorrelated with past prediction errors. This estimation is equivalent to estimating the auto correlation in forecast errors.

\(^7\)Ryngaert (2017) offers an alternative approach to estimate information rigidity alone.
The overall deviation from full information rational expectations is given by \( \gamma \), which was defined in 11 as

\[
\gamma = \frac{\sigma^2_{\nu_{t-1}} + \lambda \sigma^2_{\epsilon_{t-1}}}{\sigma^2_{\epsilon_{t-1}} + \sigma^2_{\nu_{t-1}}}
\]  

While \( \sigma^2_{\epsilon_{t-1}} \) can easily be calculated, it might not be immediately clear how to recover \( \sigma^2_{\nu_{t-1}} \), the variance of the signal noise. Acknowledging that the error term is a function of \( \nu \) makes it possible to recover this variance. Indeed, the variance of the noise is simply the variance of the error term divided by \( (1 - \gamma)^2 \).

This transformation allows to recover \( \lambda \) as well as the overall deviation from full information rational expectations. It also allows to calculate the contribution of information rigidity and uncertainty to the coefficient of interest separately. Note that without rigidity, the above estimator provides an unbiased estimate of uncertainty alone.

While the derivation so far focused on the individual level, these equations should be estimated at the aggregate level. To obtain the aggregate variables, one can simply average the above equation across individuals (or equivalently use the forecast errors of the average prediction). As mentioned above, this is necessary because the sticky information model does not allow an estimation at the individual level directly.

An important difference from the individual to the aggregate level is the gains from averaging forecasts. In particular, the individual level uncertainty is likely larger than the aggregate level one. This can also be seen by looking at the MSE at the aggregate level as shown in Figure 1.

The pattern observed for GDP is similar to other variables; the MSE is falling, the shorter the horizon and the aggregate MSE is smaller than the individual level one.\(^8\)

\(^8\)The Bloomberg Survey improvement over the SPF at the end is likely due to the different timing of the two surveys. The Bloomberg Survey has a two week lead over the SPF.
To test whether the theoretical predictions expected based on the models above also hold with data on expectations, two separate data sets on macroeconomic forecasts will be used. The first data set is the Bloomberg Survey which is used very frequently by businesses to compare economic data releases to what economists had expected beforehand (e.g. Scotti (2013) or Chen et al. (2013)). The second data set is the Survey of Professional Forecasters (SPF), which is often referred to in academic research (e.g. Carroll (2003) or Coibion and Gorodnichenko (2015)). Forecasts might not always be a reflection of expectations. However, this is in line with previous research in the area and Bürgi (2017) showed that biases due to asymmetric loss function are not a big issue for these surveys.

For both surveys, forecasts for the CPI, GDP and unemployment at a quarterly frequency will be tested for various horizons starting with current quarter forecasts up to four quarters ahead forecasts (H0-H4). The Bloomberg Survey starts for all horizons in June 2000 while the SPF sample starts in Q1 1990. The SPF reports quarterly averages while the Bloomberg Survey records end of
quarter values.\footnote{For GDP the same quarter on quarter annualized percentage changes definitions are used, unemployment is in levels and for the CPI, the SPF uses quarter on quarter percentage change while the Bloomberg Survey uses year over year percentage changes.} Estimating the coefficients for two separate surveys and three different economic variables should reduce any survey or variable specific issues.

Most individual forecasts are missing observations in both surveys and there is quite some entry and exit in the survey. Due to this the regressions were cross checked against a requirement for forecasters to have contributed more than 30 periods or 7.5 years. This is similar to the approach taken by \textit{Elliott et al.} (2008) or \textit{Capistrán and Timmermann} (2009) to resolve this issue. For the new approach outlined in equation 14, the difference between imposing and not imposing the requirement is minimal. This adjustment is thus omitted.

As survey data is likely to contain errors, the data is checked for large outliers, which are subsequently removed.\footnote{The SPF records level data for RGDP and QoQ predictions for the CPI. Because these numbers differ from the usual headline figures predicted in most other surveys, these two series might be prone to more data outliers than any of the other series.}

For the actual values, revised data is used for the CPI and unemployment, as the revisions tend to be quite small. For GDP, the third release is used.

### 5 Empirical Application

As a first step, equation 14 is estimated at the aggregate level for all variables. Table 1 presents the estimates for the auto-correlation of forecast errors. The coefficient $\gamma = \frac{\sigma^2_{t-1} + \lambda \sigma^2_{t-1}}{\sigma^2_{t-1} + \sigma^2_{t-1} + \sigma^2_{t-1}}$ captures the entire deviation from full information rational expectations and thus both information rigidity and uncertainty.

The estimates show that there is a substantial weight on old information as almost all coefficients are positive and highly significant. This implies a significant departure from full information rational expectations. Indeed, estimates show the right sign, are significant and fall within the expected range between…

$\gamma = \frac{\sigma^2_{t-1} + \lambda \sigma^2_{t-1}}{\sigma^2_{t-1} + \sigma^2_{t-1} + \sigma^2_{t-1}}$.
Table 1: Aggregate level joint rigidity and uncertainty estimates ($\gamma$)

<table>
<thead>
<tr>
<th></th>
<th>CPI BBG</th>
<th>CPI SPF</th>
<th>GDP BBG</th>
<th>GDP SPF</th>
<th>Unemployment BBG</th>
<th>Unemployment SPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0</td>
<td>0.58***</td>
<td>0.58***</td>
<td>0.61***</td>
<td>0.75***</td>
<td>0.41***</td>
<td>0.34***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.08)</td>
<td>(0.06)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>H1</td>
<td>0.96***</td>
<td>0.96***</td>
<td>0.88***</td>
<td>0.89***</td>
<td>0.58***</td>
<td>0.56***</td>
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<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>H2</td>
<td>1.00***</td>
<td>1.00***</td>
<td>0.95***</td>
<td>0.92***</td>
<td>0.68***</td>
<td>0.65***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>H3</td>
<td>1.00***</td>
<td>1.00***</td>
<td>0.97***</td>
<td>0.97***</td>
<td>0.75***</td>
<td>0.73***</td>
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<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
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HAC standard errors in brackets. * significant at 10% level, ** at 5% level and *** at 1% level. Based on the average of forecasters.

zero and one. As expected with higher uncertainty for longer horizons, the estimates become larger, the longer the horizons. This finding is also directly linked to the MSE of forecasts.

Utilizing the variance of the error term and the past prediction as well as the definition of the estimated coefficient. It is possible to calculate what part of this deviation is due to information rigidity and what part is due to uncertainty. Table 2 presents the deviation from full information rational expectations due to the information rigidity given by $\frac{\lambda \sigma_{\epsilon_{t-1}}^2}{\sigma_{\epsilon_{t-1}}^2 + \sigma_{\nu_t}^2}$. Subtracting this from the estimated coefficients in Table 1 leads to the contribution of uncertainty to this deviation.

Comparing the contribution of the information rigidity to the contribution due to uncertainty, it is immediately clear that for horizons longer than the current quarter, rigidity is negligible relative to uncertainty. The only exception is unemployment, where rigidity causes around 50% of the overall deviation in
Table 2: Estimated aggregate deviation due to $\lambda$

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBG</td>
<td>0.30</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>SPF</td>
<td>0.27</td>
<td>0.01</td>
<td>0.17</td>
</tr>
<tr>
<td>H0</td>
<td>0.03</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>H1</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>H2</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.07</td>
</tr>
<tr>
<td>H3</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
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The numbers show $\frac{\lambda \sigma_{\xi_{t-1}}^2}{\sigma_{\xi_{t-1}}^2 + \sigma_{\nu_t}^2}$, which corresponds to the additive contribution of information rigidity to the overall delay.

the shortest two horizons and then falls to around 1/3 for longer horizons. Unemployment is also the variable most likely affected by rounding, as it is only reported up to one tenth. It also appears, that rigidity plays a very small role for GDP, as the contribution is never more than 1/4. For inflation, the pattern looks different. For current quarter forecasts, the split between uncertainty and information rigidity is about 50%, but for longer horizons, the contribution from rigidity becomes negligible. One issue at longer horizons is that the coefficient capturing the overall deviation is almost equal to 1. Due to this, the variance of the estimated rigidity contribution becomes larger. However, even if 0.05 is subtracted from the inflation estimates of $\gamma$ for H1-H3 (more than two standard deviations), the contribution of information rigidity remains negligible at less than 0.15.

The low importance of rigidity at longer horizons is also due to the way rigidity enters the coefficient. While the variance of the prediction error is unlikely to become very large at long horizons, the variance of the noise around new information is likely to become very large for long horizons. This will cause the importance of rigidity to decline the longer the horizon. It thus might be
informative to calculate $\lambda$ from the equation above as well.

As Table 3 shows, there is substantial rigidity across all variables and most horizons. The negative coefficients are all within two standard deviations and many within one. Overall, this supports the notion that there is substantial rigidity as found by Coibion and Gorodnichenko (2015) as well. However its contribution to the overall deviation from full information rational expectation is clearly smaller than $\lambda$ by itself would suggest and is also limited relative to uncertainty. This is particularly the case at longer horizons.

Given the substantial rigidity found above, one should also find a similar pattern when estimating a Nordhaus type regression. As shown above, this regression will recover $\frac{\lambda}{1-\lambda}$ and also allows to estimate the rigidity coefficient $\lambda$. Table 4 shows the coefficients for the Nordhaus test at the aggregate level.

These results show the information rigidity coefficients are broadly consistent with Dovern and Weisser (2011), Lahiri and Sheng (2008) and Messina et al. (2015) who estimated both positive and negative coefficients for quarterly data. The estimates are also in the same direction as Coibion and Gorodnichenko (2015) or Dovern et al. (2015), who found positive and significant coefficients at the H0 horizon.\footnote{They did not use the same quarterly definitions as here, but similar to the definitions used here, actual data on a subset of the period predicted was already available at the time} These results are also in line with the results presented in

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<td>BBG</td>
<td>SPF</td>
<td>BBG</td>
</tr>
<tr>
<td>H0</td>
<td>0.42</td>
<td>0.39</td>
</tr>
<tr>
<td>H1</td>
<td>0.44</td>
<td>0.11</td>
</tr>
<tr>
<td>H2</td>
<td>-4.14</td>
<td>-1.27</td>
</tr>
<tr>
<td>H3</td>
<td>-6.37</td>
<td>-1.09</td>
</tr>
</tbody>
</table>

The numbers show the rigidity parameter $\lambda$. 

\[
\lambda = \frac{1}{1 - \lambda}
\]
Table 4: Information Rigidity Estimation of Average Expectations ($\frac{\lambda}{1-\lambda}$)

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPF</td>
<td>Bloomberg</td>
<td>SPF</td>
</tr>
<tr>
<td>H0</td>
<td>0.78***</td>
<td>0.35***</td>
<td>0.33***</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.11)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>H1</td>
<td>-0.30</td>
<td>0.24*</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.13)</td>
<td>(0.42)</td>
</tr>
<tr>
<td>H2</td>
<td>-0.80</td>
<td>-0.01</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.64)</td>
<td>(0.18)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>H3</td>
<td>-0.19</td>
<td>-0.02</td>
<td>0.28*</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.07)</td>
<td>(0.15)</td>
</tr>
</tbody>
</table>

Standard errors in brackets. * significant at 10% level, ** at 5% level and *** at 1% level based on HAC errors. Based on the average of forecasters with at least 30 observations.

The previous tables. They show that while rigidity is substantial by itself, its contribution to the deviation from full information rational expectations is not as large as the one due to uncertainty.

Before moving to the individual level regressions for the overall rigidity, it is important to set the baseline by estimating the $\lambda$ at the individual level correctly. As discussed above, the rigidity estimates will be biased due to the issues surrounding the sticky information model. To show this bias, first a Nordhaus type regression is run at the individual level without any corrections. Table 5 shows a Davies and Lahiri (1995) type estimation and it is indeed the case that the individual level estimates are much smaller and often negative and significant. Because of these issues, information rigidity should not be estimated by running an unmodified Nordhaus test at the individual level.

the forecast was made.
Table 5: Information Rigidity Estimation of Individual Expectations ($\lambda$)

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0</td>
<td>SPF 0.08***, (0.03)</td>
<td>Bloomberg -0.01, (0.02)</td>
<td>SPF 0.05***, (0.02)</td>
</tr>
<tr>
<td>H1</td>
<td>SPF -0.59***, (0.06)</td>
<td>Bloomberg -0.32***, (0.04)</td>
<td>SPF -0.88***, (0.02)</td>
</tr>
<tr>
<td>H2</td>
<td>SPF -0.62***, (0.07)</td>
<td>Bloomberg -0.45***, (0.04)</td>
<td>SPF -0.01, (0.02)</td>
</tr>
<tr>
<td>H3</td>
<td>SPF -0.37***, (0.08)</td>
<td>Bloomberg -0.10***, (0.01)</td>
<td>SPF -0.32***, (0.02)</td>
</tr>
</tbody>
</table>

Standard errors in brackets. * significant at 10% level, ** at 5% level and *** at 1% level based on RE panel estimation with FGLS errors. Based on forecasters with at least 30 observations.

There is however a Nordhaus type estimation introduced by Bürgi (2016), which can bring the individual level regression on par with the aggregate level ones. This approach uses the average revisions as an instrument for the individual revisions. The average revisions are a valid instrument and they remove the biases specific to the individual level. Table 6 reports the coefficients for that regression.

It is still the case that rigidity is high and significant at the individual level. However, there are some estimates, where the coefficients are no longer consistent with rational expectations based on the models above. In particular, the negative coefficients suggest that agents put more than optimal weight on new information. It could however be that agents have an incentive to make larger than optimal revisions in a Ehrbeck and Waldmann (1996) type setting. Given the positive rigidity found at the aggregate level, an explanation is needed. One
Table 6: Information Rigidity Estimation of Instrumented Individual Expectations ($\frac{\lambda}{1-\lambda}$)

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th></th>
<th>GDP</th>
<th></th>
<th>Unemployment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPF</td>
<td>Bloomberg</td>
<td>SPF</td>
<td>Bloomberg</td>
<td>SPF</td>
<td>Bloomberg</td>
</tr>
<tr>
<td>H0</td>
<td>0.86***</td>
<td>0.33***</td>
<td>0.16***</td>
<td>0.31***</td>
<td>0.25***</td>
<td>0.35***</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>H1</td>
<td>-0.31***</td>
<td>0.20***</td>
<td>0.03</td>
<td>0.27**</td>
<td>0.23***</td>
<td>0.72***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.07)</td>
<td>(0.29)</td>
<td>(0.13)</td>
<td>(0.03)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>H2</td>
<td>-0.96***</td>
<td>-0.07</td>
<td>0.20*</td>
<td>0.60***</td>
<td>0.29***</td>
<td>0.92***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.08)</td>
<td>(0.12)</td>
<td>(0.21)</td>
<td>(0.03)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>H3</td>
<td>0.21</td>
<td>0.00</td>
<td>0.26***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.08)</td>
<td>(0.03)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in brackets. * significant at 10% level, ** at 5% level and *** at 1% level based on two stage least squares RE panel estimation with FGLS errors, where the average revision is used as an instrument. Based on forecasters with at least 30 observations.

such explanation could be that agents update their predictions marginally most of the time and occasionally make very large revisions. The large revisions are an over adjustment as defined in Nordhaus (1987). Averaging the predictions across individuals would lead the occasional large revisions to have less importance leading to the empirically measured rigidity.

The overall deviations from full information rational expectations can also be done at the individual level. At the individual level, the overall deviations from full information rational expectations are extensive as well, as Table 7 reports.

At the first look, both the aggregate coefficients and the individual coefficients look the same; the coefficients are between zero and one, highly significant
Table 7: Individual level joint rigidity and uncertainty estimates ($\gamma$)

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BBG SPF</td>
<td>BBG SPF</td>
<td>BBG SPF</td>
</tr>
<tr>
<td>H0</td>
<td>0.58***</td>
<td>0.56***</td>
<td>0.62***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>H1</td>
<td>0.93***</td>
<td>0.93***</td>
<td>0.87***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>H2</td>
<td>0.96***+</td>
<td>0.97***+</td>
<td>0.93***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>H3</td>
<td>0.96***+</td>
<td>0.96***+</td>
<td>0.95***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

HAC standard errors in brackets. * significant at 10% level, ** at 5% level and *** at 1% level based on panel estimation. + significantly different to the aggregate level estimation at the 5% level.

and become larger, the longer the horizon. At the second look, one might notice that there are four longer horizon cases, where the estimates at the aggregate and individual level are different. Indeed, the aggregate level deviation appears to be larger than the individual one. This is likely due to the downward bias from the sticky information model. To remove the bias, the estimates from the instrumented Nordhaus regression in Table 6 can be used.

Table 8 reports the contribution of information rigidity to the overall bias deviation from full information rational expectations taking the rigidity coefficients from Table 6.

The results all show lower contributions than the aggregate level, except for the unemployment rate in the Bloomberg survey. These contributions are larger than the ones without this adjustment, particularly, all negative values.
Table 8: Estimated individual level delay due to $\lambda$

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BBG</td>
<td>SPF</td>
<td>BBG</td>
</tr>
<tr>
<td>H0</td>
<td>0.05</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>H1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>H2</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>H3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The numbers show $\frac{\lambda_2 \sigma_{t-1}^2}{\sigma_{t-1}^2 + \sigma_{t-1}^2}$, which corresponds to the additive contribution of information rigidity to the overall delay using the instrumented $\lambda$ from Table 6.

are removed.

Overall, the results presented both at the aggregate and individual level suggest that while rigidity is substantial, its contribution to the overall deviation from full information rational expectations is rather limited.

6 Conclusion

This paper introduces a new approach to estimate the deviation from full information rational expectations and the resulting delay as measured by the auto-correlation of forecast errors. This new approach allows to pinpoint the contribution to the delay from information rigidity and from uncertainty.

The estimates show that while both the overall deviation and the extent of information rigidity is extensive, the contribution of information rigidity to this deviation is rather limited, particularly for longer horizons. In turn, the contribution of uncertainty is large, particularly at longer horizons. The implication from this for policymakers is that they should focus on influencing with the uncertainty arising from policy actions. Influencing uncertainty might be easier than interacting with information rigidity and also has a much larger impact.
Crawford and Sobel (1982) showed one example how and why policy makers might increase or decrease uncertainty.

This results hold both at the aggregate and individual level although rigidity appears to be smaller at the individual level. Further research might be able to pinpoint if this is due to agents making minimal or no updates to their forecast most of the time and infrequently very large updates to the forecasts.

This paper also provides a new link from the information rigidity literature to uncertainty as it was shown that the contribution of rigidity to the overall deviation from full information rational expectations is linked to uncertainty. The paper also links uncertainty to the actions of agents; higher uncertainty will cause agents to optimally put more weight on past information. This provides new insights into the interaction between agents and uncertainty, which can be utilized in future research to better capture if and by how much agents react to uncertainty. It can also lead to new uncertainty indexes, which assess when agents react to higher uncertainty and when not by comparing the weight put on old information relative to new information. This allows to create a measure of ex-ante uncertainty, even if the actual realization is needed to calculate.
References


The Nordhaus (1987) test checks weak efficiency by checking if forecast revisions can explain current forecast errors.

It assumes that forecasts are a weighted average of new and old information and tests whether the new forecasts put the optimal weight on new information and is thus weakly efficient. Any forecast $F_{t,t-i}$ made in period $t-i$ for period $t$ is thus the actual value $A_t$ plus some error $\nu_{t-i}$ with variance $\sigma_\nu$. At the same time, it is a weighted average of the old forecast $F_{t,t-i-1}$ and some new information with error $\mu_{t-i}$. That is

$$F_{t,t-i} = A_t + \nu_{t-i} = \gamma F_{t,t-i-1} + (1-\gamma)(A_t + \nu_{t-i}), \quad (16)$$

where $\gamma$ is the weight put on old information. It is assumed, that $\nu_{t-i}$ and $\mu_{t-i}$ are independent from each other. This allows the regression specification

$$A_t - F_{t,t-i} = \alpha + \beta(F_{t,t-i} - F_{t,t-i-1}) + \varepsilon_t, \quad (17)$$

where under weak efficiency $\alpha = 0$ and $\beta = 0$.\(^\text{12}\) The estimator for the coefficient of interest then becomes

\[\hat{\beta} = \frac{\text{cov}[A_t - F_{t,t-i}, F_{t,t-i} - F_{t,t-i-1}]}{\text{var}[F_{t,t-i} - F_{t,t-i-1}]} \quad (18)\]

\[= \frac{\text{cov}[-\gamma \mu_{t-i-1} - (1-\gamma)\nu_{t-i}, (\gamma-1)\mu_{t-i-1} + (1-\gamma)\nu_{t-i}]}{\text{var}[\gamma-1)\mu_{t-i-1} + (1-\gamma)\nu_{t-i}]} \quad (19)\]

\[= \frac{\text{cov}[-\gamma \mu_{t-i-1} - (1-\gamma)\nu_{t-i}, (1-\gamma)(\nu_{t-i} - \mu_{t-i-1})]}{(1-\gamma)^2\text{var}[\nu_{t-i} - \mu_{t-i-1}]} \quad (20)\]

\[= \frac{\gamma(1-\gamma)\sigma^2_\mu - (1-\gamma)^2\sigma^2_\nu}{(1-\gamma)^2(\sigma^2_\mu + \sigma^2_\nu)} \quad (21)\]

\[= \frac{\gamma\sigma^2_\mu - (1-\gamma)\sigma^2_\nu}{(1-\gamma)(\sigma^2_\mu + \sigma^2_\nu)} \quad (22)\]

\(^\text{12}\)There is an equivalent specification with the same assumptions using only revisions of the form $F_{t,t-i} - F_{t,t-i-1} = \alpha + \beta(F_{t,t-i-1} - F_{t,t-i-2}) + \varepsilon_t$. That specification has the benefit or not requiring the actual value of the underlying variable and is also detailed in Nordhaus (1987). It has the same $\hat{\beta}$ as the approach described above, except that the variances correspond to $\mu_{t-i-2}$ and $\nu_{t-i-1}$

---

A The Nordhaus Test

The Nordhaus (1987) test checks weak efficiency by checking if forecast revisions can explain current forecast errors.

It assumes that forecasts are a weighted average of new and old information and tests whether the new forecasts put the optimal weight on new information and is thus weakly efficient. Any forecast $F_{t,t-i}$ made in period $t-i$ for period $t$ is thus the actual value $A_t$ plus some error $\nu_{t-i}$ with variance $\sigma_\nu$. At the same time, it is a weighted average of the old forecast $F_{t,t-i-1}$ and some new information with error $\mu_{t-i}$. That is

$$F_{t,t-i} = A_t + \nu_{t-i} = \gamma F_{t,t-i-1} + (1-\gamma)(A_t + \nu_{t-i}), \quad (16)$$

where $\gamma$ is the weight put on old information. It is assumed, that $\nu_{t-i}$ and $\mu_{t-i}$ are independent from each other. This allows the regression specification

$$A_t - F_{t,t-i} = \alpha + \beta(F_{t,t-i} - F_{t,t-i-1}) + \varepsilon_t, \quad (17)$$

where under weak efficiency $\alpha = 0$ and $\beta = 0$.\(^\text{12}\) The estimator for the coefficient of interest then becomes

\[\hat{\beta} = \frac{\text{cov}[A_t - F_{t,t-i}, F_{t,t-i} - F_{t,t-i-1}]}{\text{var}[F_{t,t-i} - F_{t,t-i-1}]} \quad (18)\]

\[= \frac{\text{cov}[-\gamma \mu_{t-i-1} - (1-\gamma)\nu_{t-i}, (\gamma-1)\mu_{t-i-1} + (1-\gamma)\nu_{t-i}]}{\text{var}[\gamma-1)\mu_{t-i-1} + (1-\gamma)\nu_{t-i}]} \quad (19)\]

\[= \frac{\text{cov}[-\gamma \mu_{t-i-1} - (1-\gamma)\nu_{t-i}, (1-\gamma)(\nu_{t-i} - \mu_{t-i-1})]}{(1-\gamma)^2\text{var}[\nu_{t-i} - \mu_{t-i-1}]} \quad (20)\]

\[= \frac{\gamma(1-\gamma)\sigma^2_\mu - (1-\gamma)^2\sigma^2_\nu}{(1-\gamma)^2(\sigma^2_\mu + \sigma^2_\nu)} \quad (21)\]

\[= \frac{\gamma\sigma^2_\mu - (1-\gamma)\sigma^2_\nu}{(1-\gamma)(\sigma^2_\mu + \sigma^2_\nu)} \quad (22)\]

\(^\text{12}\)There is an equivalent specification with the same assumptions using only revisions of the form $F_{t,t-i} - F_{t,t-i-1} = \alpha + \beta(F_{t,t-i-1} - F_{t,t-i-2}) + \varepsilon_t$. That specification has the benefit or not requiring the actual value of the underlying variable and is also detailed in Nordhaus (1987). It has the same $\hat{\beta}$ as the approach described above, except that the variances correspond to $\mu_{t-i-2}$ and $\nu_{t-i-1}$
where $\sigma^2_\mu$ is the variance of $\mu_{t-1}$ and $\sigma^2_\nu$ is the variance of $\nu_{t-1}$. Under weak efficiency, agents utilize the Bates and Granger (1969) optimal weights that minimize the MSE. The optimal weights are equivalent to inverse variance weights, assuming the signal is independent from past predictions. That is

$$\gamma = \frac{1}{\sigma^2_\nu + \frac{1}{\sigma^2_\mu}}.$$  \hfill (23)

Plugging these weights into equation 22 will render the numerator equal to 0 and the estimate should be equal to 0 as well. If $\gamma$ is larger than the optimal weight, the coefficient will become positive. This implies that agents put more than efficient weight on old information and under adjust to new information. Examples of this is described above, be it the sticky information model or rounding. If $\gamma$ is smaller than the optimal weight, the coefficient will become negative. This implies that agents put less than efficient weight on old information and over adjust to new information. Examples of this could be due to a Ehrbeck and Waldmann (1996) type setup, or agents assuming that the new information was more valuable than it actually is.

13 Due to the independence assumption between $\mu_{t-1}$ and $\nu_{t-1}$, all the covariance terms disappear.
B Kalman implementation with uncertainty

The Kalman (1960) filter is used to track an underlying process, which cannot be accurately measures. It usually requires the exact knowledge of the underlying process and was originally used for navigation. There, the determinants of the position like speed and direction are known but the exact position cannot be measured. Assuming however that the prediction made for period t in period t-i is the optimal prediction based on the Kalman filter, less stringent assumptions are necessary for an observer. Assume that agents measure a set of variables every period, which provide information about the underlying variable in period t. This information is not perfectly accurate and thus only provides a noisy signal about the actual realization $A_t$ of the form

$$x_{jt-i} = A_t + \nu_{jt-i}, \quad (24)$$

where $\nu_{jt-i}$ is an iid error with variance $\sigma^2\nu_{jt-i}$. Given this noisy signal, every agent can then produce an optimal prediction with the Kalman filter. Let $F_{jt,t-1-i}$ denote the prediction made in period t-1-i for period t with variance $\sigma^2\varepsilon_{t-1-i}$. Since the past prediction is observed and the noisy signal is about the future state of the variable, the assumption about the exact mechanism of the underlying variable becomes less important. Indeed, this setup is the same if the underlying variable is an AR(1), an AR(p), a random walk or only depends on other (observable) variables. This setup is thus very flexible.

In period t-i, the noisy signal is observed which is independent from the past prediction by assumption. These two signals are then optimally weighted according to the inverse of their variances (i.e. the Bates and Granger (1969) optimal weights) to produce the optimal new prediction. The weight put on the new information then becomes

$$1 - G = \frac{\sigma^2\nu_{t-1-i}}{\sigma^2\nu_{t-1-i} + \sigma^2\varepsilon_{t-1-i}}, \quad (25)$$

which is also called the Kalman gain and is directly related to the uncertainty.
around the past prediction and the noise in the signal. In this implementation of the Kalman filter, there is no information rigidity and it is only driven by uncertainty. The new prediction then becomes

\[ F_{jt,t-i} = (1 - G)x_{jt-i} + GF_{jt,t-1-i} \]  

(26)

for any desired horizon \( i \). This can be rearranged to

\[ A_t - F_{jt,t-i} = \frac{G}{1 - G}(F_{jt,t-i} - F_{jt,t-i-1}) - \nu_{jt-i} \]  

(27)

to have the same format as in the discussion above, where the error term is correlated with the revision. Note that as shown in Appendix A, the coefficient of interest will be equal to 0, as the Kalman filtered series optimally weights old and new information.